Mr. Michael T. Davis WLPCS Calculus	Sections 1.3 & 1.5 Practice Quiz September 26, 2017

Name: Solutions / Answers

Directions: Try each problem without your calculator, then use your calculator ONLY IF you feel you have truly tried everything possible without your calculator.

1. Determine if each function is a one-to-one function. Answer with "yes" or "no" and explain why or why not.

a. $f(x) = \ln x$

f(x) is one-to-one because for each y-value there corresponds exactly one x-value

b. $g(x) = |x|$

g(x) is not one-to-one since there are y-values that correspond with two x-values.
e.g. $f(4) = 4$ and $f(-4) = 4$

c. $k(x) = \frac{1}{x}$

k(x) is one-to-one since for each y-value there corresponds exactly one x-value

d. $L(x) = \sin x$

L(x) is not one-to-one since there are y-values that correspond to many x-values.
e.g. $L(\frac{\pi}{6}) = \frac{1}{2}$ and $L(\frac{5\pi}{6}) = \frac{1}{2}$ and $L(\frac{13\pi}{6}) = \frac{1}{2}$
etc...

2. Given the function $T(x) = 2x - 6$, determine the rule for the inverse function $T^{-1}(x)$.

$y = 2x - 6$
$x = 2y - 6$
$2y = x + 6$
$y = \frac{1}{2}x + 3$

$T^{-1}(x) = \frac{1}{2}x + 3$

3. Given the function $f(x) = e^x$, determine the rule for the inverse function $f^{-1}(x)$.

$y = e^x$
$x = e^y$
$\ln x = y$
$y = \ln x$

$f^{-1}(x) = \ln x$

4. Given the function $g(x)=\sqrt{x-4}$, determine the rule for the inverse function $g^{-1}(x)$.

$y=\sqrt{x-4}$

$x=\sqrt{y-4}$

$x^2=y-4$

$y=x^2+4,\ x\geq 0$

$g^{-1}(x)=x^2+4,\ x\geq 0$

5. The two functions $f(x)=\frac{3}{2}x-6$ and $g(x)=\frac{2}{3}x+4$ are inverses. Determine the value of the composite function $f(g(x))$.

$f(\frac{2}{3}x+4)=\frac{3}{2}(\frac{2}{3}x+4)-6$

$=x+6-6$

$=x$

$f(g(x))=x$

which tells us that $f(x)$ and $g(x)$ may be inverse functions.

6. Identify all functions that have inverse functions?

a. $T(x)=9-x$ yes, $T^{-1}(x)=-x+9$

b. $f(x)=\sqrt{9-x}$ yes, $f^{-1}(x)=9-x^2,\ x\geq 0$

c. $g(x)=\sqrt{9-x^2}$ NO, since $g(x)$ is not one-to-one

d. $P(x)=x^2$ NO, since $P(x)$ is not one-to-one

e. $k(x)=x^2$ *if* $x\geq 0$ yes, since $K(x)$ is one-to-one

f. $W(x)=\ln(x-1)$ yes, $W^{-1}(x)=e^x+1$

g. $L(x)=e^{x+3}$ Yes, $L^{-1}(x)=\ln(x)-3$

7. A one-to-one function $y=f(x)$ is such that $f(14)=3$. Determine the value of $f^{-1}(3)$

$f^{-1}(3)=14$

8. Solve the equation $\ln y=3t-2$ for y.

$e^{3t-2}=y$

$y=e^{3t-2}$

9. Solve the equation $5^t=20$ for t.

$\ln 5^t=\ln 20$

$t\ln 5=\ln 20$

$t=\dfrac{\ln 20}{\ln 5}$ or $t=\log_5 20$

10. Solve the equation $e^{2t}=10$ for t.

$\ln e^{2t}=\ln 10$

$2t\ln e=\ln 10$

$2t=\ln 10$

$t=\dfrac{\ln 10}{2}$

$t=\frac{1}{2}\ln 10$

11. Solve the equation $\log_m 81=4$ for m.

$m^4=81$

$m=3$

$m\neq -3$ since bases cannot be negative

12. Solve the equation $\log_2 k=-4$ for k.

$2^{-4}=K$

$\dfrac{1}{2^4}=K$

$\dfrac{1}{16}=K$

13. Solve the equation $\log_2 t + \log_2(t-6) = 4$ for t.

$\log_2(t^2-6t) = 4$

$2^4 = t^2-6t$

$0 = t^2-6t-16$

$0 = (t-8)(t+2)$

$t=8 \quad t \neq -2$

$\{8\}$

14. Solve the inequality $\log x > 0$ for x.

$\{x>1\}$

15. True or False: $\log_m(45) = \log_m 5 + \log_m 9$

16. True or False: $\log_a\left(\frac{17}{11}\right) = \log_a 17 - \log_a 11$

17. True or False: $\log_b(x^5) = 5\log_b(x)$

18. True or False: $\frac{\log_n 13}{\log_n 7} = \log_n 13 - \log_n 7$

19. True or False: $\ln\frac{w}{\sqrt{v}} = \ln w - \frac{1}{2}\ln v$

20. True or False: $\log_a x = \frac{\ln x}{\ln a}$

21. Evaluate or simplify the expression $8^{\log_8 15}$ $= 15$

22. Evaluate or simplify the expression $\log_9\left(9^y\right)$. $= y$

23. State the domain and range of the function $f(x) = \ln x$

D: $(0, \infty)$

R: $(-\infty, \infty)$

24. State the domain and range of the function $f(x) = \ln x + 3$

D: $(0, \infty)$

R: $(-\infty, \infty)$

25. State the domain and range of the function $f(x) = \ln(x+3)$

D: $(-3, \infty)$

R: $(-\infty, \infty)$

26. State the domain and range of the function $f(t) = e^t$

D: $(-\infty, \infty)$

R: $(0, \infty)$

27. State the domain and range of the function $f(t)=e^t-3$

$D: (-\infty, \infty)$

$R: (-3, \infty)$

28. State the domain and range of the function $f(t)=e^{t-3}$

$D: (-\infty, \infty)$

$R: (0, \infty)$

29. The half-life of a certain radioactive substance is 120 days. There are 10 grams initially. Express the amount of substance remaining as a function of time t, where t is in days. Determine how many grams remain after 175 days.

$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{k}}$

$A(175) = 10\left(\frac{1}{2}\right)^{\frac{175}{120}}$

$A(175) \approx 3.639$ grams

30. The half-life of a certain radioactive substance is 120 days. There are 10 grams initially. Express the amount of substance remaining as a function of time t, where t is in days. In how many days will there be 2.5 grams remaining?

$A(t) = A_0\left(\frac{1}{2}\right)^{t/k}$

$2.5 = 10\left(\frac{1}{2}\right)^{\frac{t}{120}}$

$\frac{2.5}{10} = \left(\frac{1}{2}\right)^{\frac{t}{120}}$

$0.25 = \left(\frac{1}{2}\right)^{\frac{t}{120}}$

$\ln(0.25) = \frac{t}{120}\ln\left(\frac{1}{2}\right)$

$\frac{\ln(0.25)}{\ln\left(\frac{1}{2}\right)} = \frac{t}{120}$

$\frac{120\ln(0.25)}{\ln(0.5)} = t$

$t \approx 240$ days

31. The half-life of a certain radioactive substance is 120 days. There are 10 grams initially. Express the amount of substance remaining as a function of time t, where t is in days. In how many days will there be 1.75 grams remaining?

$1.75 = 10\left(\frac{1}{2}\right)^{\frac{t}{120}}$

$0.175 = \left(\frac{1}{2}\right)^{\frac{t}{120}}$

$\ln(0.175) = \frac{t}{120}\ln(0.5)$

$\frac{120\ln(0.175)}{\ln(0.5)} = t$

$t \approx 301.749$ days

32. Determine the future value of $500 invested for 15 years in an interest bearing account with an annual interest rate of 5.75% compounded monthly.

$$A(t) = A_0\left(1+\frac{r}{12}\right)^{12t}$$
$$A(15) = 500\left(1+\frac{.0575}{12}\right)^{(12)(15)}$$
$$A(15) \approx \$1{,}182.10$$

33. Determine the future value of $500 invested for 15 years in an interest bearing account with an annual interest rate of 5.75% compounded continuously.

$$A(t) = A_0 e^{rt}$$
$$A(15) = 500\, e^{0.0575(15)}$$
$$A(15) \approx \$1{,}184.54$$

34. Determine how much time is required for a $500 investment to double in value if the interest is earned at a rate of 5.75% compounded weekly.

$$1000 = 500\left(1+\frac{0.0575}{52}\right)^{52t} \qquad t \approx 12.06 \text{ years}$$
$$2 = \left(1+\frac{0.0575}{52}\right)^{52t}$$
$$\ln 2 = 52t \ln\left(1+\frac{0.0575}{52}\right)$$
$$\frac{\ln 2}{52\ln\left(1+\frac{0.0575}{52}\right)} = t$$

35. Determine how much time is required for a $500 investment to triple in value if the interest is earned at a rate of 5.75% compounded continuously.

$$1500 = 500\, e^{0.0575t}$$
$$3 = e^{0.0575t}$$
$$\ln 3 = \ln e^{0.0575t}$$
$$\ln 3 = 0.0575t$$
$$\frac{\ln 3}{0.0575} = t$$
$$t \approx 19.11 \text{ years}$$